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Original Article

Empirical Derivation of Capital Asset Pricing Method (CAPM) Approach in Option Pricing

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Abstract

This paper investigates how the classical equilibrium asset-pricing framework embodied in the Capital Asset Pricing Model (CAPM) can be extended to the valuation of derivative securities, specifically European-style options. While traditional option pricing models (e.g., Black-Scholes model) rely on no-arbitrage and risk-neutral valuation, they do not explicitly incorporate systematic risk (beta) or investor planning horizon. We develop a unified theoretical model that embeds the option payoff's covariance with the market portfolio and allows for differences between investor horizon and option maturity. We then describe an empirical strategy to test whether options on higher-beta underlying's command higher required returns and whether pricing deviations from standard models are systematically related to underlying beta, time to maturity, and moneyness. Our findings provide preliminary support for the CAPM-based option pricing logic: underlying systematic risk matters for option valuation, and the investor horizon effect is non-negligible in less idealized markets. We discuss implications for option traders, risk managers and researchers, especially in markets where replication is imperfect and investor horizons are heterogeneous.

Keywords

CAPM, Option Pricing, Systematic Risk, Option Beta, Planning Horizon, Market Incompleteness.

Introduction

Option markets serve critical roles in modern financial systems: they allow market participants to hedge exposures, express views on volatility and direction, structure exotic pay-offs, and facilitate risk transfer. Traditional option-valuation frameworks — most notably the Black–Scholes model — rely on **no-arbitrage** logic and a risk-neutral pricing measure, rather than invoking explicit investor risk preferences or the equilibrium trade-off between risk and return. In such models, the underlying asset is assumed to follow a specified stochastic process (e.g., geometric Brownian motion), and pricing is decoupled from how investors require compensation for bearing systematic risk.

In contrast, the Capital Asset Pricing Model (CAPM) offers a canonical equilibrium approach in which the expected return on any asset is determined by its covariance (beta) with the market portfolio and the market's risk premium. Specifically:

$$E[R_i] = R_f + \beta_i (E[R_m] - R_f).$$

The question then arises: can the CAPM framework be extended to price options? Doing so would embed the systematic risk characteristics of options (which are nonlinear functions of the underlying) into valuation, thereby bridging derivative pricing and classical asset-pricing theory.

This paper pursues that aim on three fronts. First, we develop a unified theoretical framework that derives an option-pricing expression using CAPM-type logic (i.e., expected-payoff minus risk premium tied to covariance with the market) and show how this nests the standard risk-neutral model under special (complete-market) conditions. Second, we bring empirical evidence to bear by investigating whether options written on underlying assets with higher systematic risk (higher beta) exhibit systematically different required returns and valuation deviations compared to low-beta underlying's. Third, we discuss practical implications for derivative traders and risk managers, especially in markets (such as emerging ones) where market completeness and deep liquidity may not hold.

The contributions of this study are: (i) to provide a clear exposition of the CAPM-based option pricing logic; (ii) to test empirically whether the risk-pricing implications of CAPM carry through to options; and (iii) to draw out how deviations from the standard no-arbitrage approach (e.g., investor horizon, market incompleteness) matter in real-world option markets.

Literature Review

A. CAPM Background

The CAPM, formulated in the 1960s (see William F. Sharpe (1964) among others), posits that only the systematic (market-related) risk of an asset matters in determining its expected return. The key formula, $E[R_i] = R_f + \beta_i(E[R_m] - R_f)$, emphasises that indiscriminate (idiosyncratic) risk is diversifiable and thus not priced. The model rests on strong assumptions: mean-variance efficient investors, a single-period horizon, homogeneous expectations, and ability to borrow/lend at the risk-free rate.

Despite its elegant simplicity, the empirical performance of CAPM has been challenged: anomalies such as size, value, momentum effects (e.g., as documented by Eugene F. Fama and Kenneth R. French) suggest that additional risk factors beyond market beta may help explain cross-sectional returns. Moreover, critiques such as Richard Roll's highlight the empirical unobservability of the true market portfolio, which complicates direct tests of CAPM.

Nevertheless, CAPM remains foundational in finance — not only as a benchmark but as a building block for multi-factor models and for understanding systematic risk in asset valuation.

B. Option Pricing Theory

The foundational work of Black & Scholes (1973) introduced a closed-form formula for European call options, assuming a complete market, no arbitrage, continuous trading, lognormal underlying asset returns, and constant volatility. Under those assumptions, options are redundantly priced by replication: one can hedge the option by dynamically trading the underlying and the risk-free asset.

In more realistic settings, market incompleteness, investor preferences, stochastic volatility, jumps, and other imperfections mean that the pure risk-neutral pricing logic may not fully capture observed option prices. Researchers have extended the basic models via binomial lattices, finite-difference PDE methods, Monte Carlo simulation, and more general equilibrium frameworks.

C. Bridging CAPM and Option Pricing

There is an emerging stream of literature that seeks to apply equilibrium asset-pricing logic (such as CAPM) to derivative securities. For example, Riccardo Cesari & D'Adda (2001) propose a "simple approach" in which asset valuation is broken into two characteristics (expected value and expected variability) and show that this logic leads to a CAPM-style pricing rule, which can then be applied to options. In their framework, under normal-distribution assumptions, the resulting pricing expression coincides with the Black–Scholes price.

In a more explicit treatment, Sven Husmann & Neda Todorova (2011) derive option pricing equations under CAPM logic in incomplete lognormal markets. They show that investor's planning horizon matters: if the investor horizon differs from the option maturity, pricing deviates from the standard risk-neutral price. Their valuation formula is of the form

$$C_0 = \frac{E[C_t] - \lambda \operatorname{Cov}(C_t, R_m)}{(1+r)^h},$$

Where the second term captures the risk premium for covariance with the market portfolio.Under complete markets or when the planning horizon equals the maturity,this collapses to the standard Black-Scholes price.

Moreover,the work by Joel M.Vanden (2004) studying "Options Trading and the CAPM" finds that options written on the market portfolio are not redundant in equilibrium when investors face non-negativity constraints and market imperfections,thus making the CAPM logic relevant for derivatives.

The literature identifies several important insights: (i) options inherit systematic risk via their underlying exposures, meaning their expected returns and pricing should reflect underlying beta and covariance; (ii) investor horizon, market completeness, and underlying correlations with the market influence option valuation beyond pure volatility; (iii) empirical work remains comparatively sparse, especially in less-liquid or emerging markets, leaving open the question of how well CAPM-based option pricing performs in practice.

In sum, bridging CAPM and option pricing offers both theoretical richness and practical relevance — but also introduces complexities (e.g., computing option betas, dealing with market incompleteness, estimating horizons) that warrant careful empirical investigation.

Theoretical Framework

A. Model Setup and Notation

Let us consider an underlying asset S(e.g., a stock) whose current price is S_0. We assume that by time T(option maturity) the price will be S_T. Let there be a risk-free asset (or rate) with return $R_f$over the period. Let the market (or market portfolio) return over the same horizon be R_m, with expected value $E[R_m]$and variance $\mathrm{Var}(R_m)$.

We adopt the framework of the classic equilibrium model Capital Asset Pricing Model (CAPM) which states:

$$E[R_i] = R_f + \beta_i \left(E[R_m] - R_f\right) \qquad (1)$$

Where

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}.$$

This holds for an asset i.

In our setting, the option is a derivative security written on S. Denote the option (for example, a European call) payoff at maturity T by

$$C_T = \max(S_T - K, 0), \qquad (2)$$

Where K is the strike price. The current option price is C_0.

Because the option is a nonlinear payoff on the underlying S, its risk characteristics (covariance with the market) will not simply equal the underlying's beta. We must derive how to embed the option into the CAPM-type pricing logic.

We also allow for the possibility that the investor's planning horizon hand the option's time-to-maturity $\tau = T - 0$differ (so possibly $h \le \tau$or $h > \tau$)—a feature emphasized in the literature. (See Sven Husmann & Neda Todorova (2011)

Methodology and Discussion

A. Deriving the Option Pricing under CAPM Logic

Following the exposition in Cesari & D'Adda (2001) and Husmann & Todorova (2011), we outline the key steps.

B. Characteristic-based valuation (Cesari & D'Adda)

Cesari and D'Adda show that, under a mean-variance world (normal distributions or quadratic utility), the price $P_X$of any asset $X$with payoff random variable X(at horizon T) can be written as:

$$P_X = P_1\, E[X] \; - \; P_2\, \mathrm{Cov}(X, M), \qquad (3)$$

Where Mdenotes the payoff of the market portfolio and $P_1, P_2$are factor-prices for the two characteristics (expected payoff and covariance/variance contribution).

With appropriate manipulation, this form leads to the CAPM relation for an asset's expected return. The authors then apply the same logic to option payoffs $X = C_T$. Under normal-distribution assumptions they show that the resulting option-price formula coincides with the standard Black–Scholes model price.

C. CAPM-Option Pricing (Husmann & Todorova)

Husmann & Todorova extend the approach by explicitly modelling the investor's planning horizon $h$and allowing the markets to be incomplete (so perfect replication may not hold). They show that for an option held to horizon h, the price can be written as:

$$C_0 = \frac{E[C_h] \; - \; \lambda\, \mathrm{Cov}(C_h, R_m)}{(1+R_f)^h}, \qquad (4)$$

Where $\lambda$is the market price of risk over horizon h.

Important features:

- $E[C_h]$is the expected value (under the real-world measure) of the option's value at horizon h.
- $\mathrm{Cov}(C_h, R_m)$captures the systematic risk of the option payoff with respect to the market.
- The denominator $(1+R_f)^h$discounts at the risk-free rate for horizon h.
- If markets are complete and $h = \tau$(i.e., investor horizon equals option maturity) and if risk-neutral valuation holds (i.e., $\lambda = 0$), then the formula reduces to the Black–Scholes price.

They further derive closed-form expressions for $E[C_h]$and $\mathrm{Cov}(C_h, R_m)$under the lognormal assumptions of underlying and market returns. For example (excerpting from their notation):

$$E[C_h] = \text{(a function of) } S_0, \mu_s, \sigma_s, \rho\, \sigma_s \sigma_m, \tau \quad (5)$$

and

$$\mathrm{Cov}(C_h, R_m) = \mathrm{e}^{\left(\mu_m + \frac{1}{2}\sigma_m^2\right)h + r(h-\tau)}\, C[\rho\, \sigma_s \sigma_m\, \tau,\, \tau], \quad (6)$$

with $C[\cdot]$denoting a function derived from the option's payoff integrals.

Thus, the option price reflects not only the volatility of the underlying, the time-to-maturity, the strike etc., but also how the option's payoff co-moves with the market (systematic risk) and the investor's planning horizon

D. Implications for the Valuation and Empirical Hypotheses

From the foregoing theory one derives several implications:

- Because the option's payoff has systematic risk (via the underlying's correlation with the market), its covariance with the market must be priced via β-type exposure in the CAPM-option framework. Therefore, higher underlying beta and higher correlation with the market should lead to higher required returns and thus higher option prices (other things equal).
- Differences between investor horizon hand option maturity τmatter for pricing: when $h < \tau$, the investor sells the option before maturity, or their effective horizon is shorter; this leads to a different pricing kernel and thus a different price than the standard risk-neutral model.
- In incomplete markets or when perfect replication is impossible, the traditional no-arbitrage price (Black–Scholes) may not hold; the CAPM-option model suggests adjustments via systematic risk and horizon effects.

- Empirically testable hypotheses thus include:

1. H_1: Options written on underlying's with higher market beta will yield higher average returns (or higher required returns) than those on underlying's with lower beta, controlling for volatility, moneyness, maturity.
2. H_2 : Pricing deviations (observed option price minus Black–Scholes price) will be positively related to underlying beta and to the difference between investor horizon and maturity.
3. H_3: The option's effective beta (with respect to the market) can be expressed as a function of underlying beta and correlation parameters; therefore, estimation of option beta should uncover a systematic relation with underlying beta as predicted by Husmann & Todorova (2011)

E. Derivation of the CAPM-Option Pricing Formula

a) Assumptions & Notation

- Underlying asset price today S_0; time to maturity of the option is τ.
- Investor's planning horizon is h. The horizon may equal τ, may be shorter or longer.
- Underlying's return and market portfolio return are assumed jointly log-normal (or equivalently underlying price has log-normal dynamics, market has log-normal dynamics). (Husmann & Todorova).
- Instantaneous parameters: underlying asset has expected instantaneous return μ_s, volatility σ_s; market portfolio has expected instantaneous return μ_m, volatility σ_m; correlation between underlying and market returns is ρ. (H&T).
- Define $C[\, \rho\, \sigma_s\, \sigma_m\, \tau,\, \tau]$to mean the Black–Scholes value of a call whose underlying's volatility term is replaced by $\rho\, \sigma_s\, \sigma_m$over horizon τ. (H&T use this notation)
- Define $C[0, \tau]$to mean the Black–Scholes value of the call under zero correlation (or equivalently risk-neutral underlying with volatility σ_s). (H&T).

F. Expected Payoff of the Option at Horizon

Husmann & Todorova derive that the expected value of the option at horizon $h$can be expressed as (in their notation)

$$E[\ \tilde{C}_h \mid \mathcal{F}_0\ ] = e^{\,r\,(h-\tau)}\,C[0,\tau]\ , \tag{7}$$

under the case when $h \geq \tau$. (Equation 12 in their paper)
Here $r$is the risk-free rate, and the exponential term discounts (or grows) the pay-off from maturity back to horizon.

G. Covariance with Market Portfolio Return

They then compute

$$E[\,\tilde{C}_h \cdot \tilde{R}_m \mid \mathcal{F}_0] \quad = e^{\left(\mu_m+\frac{1}{2}\sigma_m^2\right)h+r\,(h-\tau)}\,C[\rho\,\sigma_s\,\sigma_m\,\tau,\,\tau] \tag{8}$$

$$\mathrm{Cov}(\tilde{C}_h, \tilde{R}_m \mid \mathcal{F}_0) \quad = e^{\left(\mu_m+\frac{1}{2}\sigma_m^2\right)h+r\,(h-\tau)}(C[\rho\,\sigma_s\,\sigma_m\,\tau,\,\tau] - C[0,\tau]) \tag{9}$$

H. Valuation Equation under CAPM Logic

Under CAPM logic, the price of the option today is given by:

$$C_0 = \frac{E[\,\tilde{C}_h]\ -\ \lambda_h\,\mathrm{Cov}(\tilde{C}_h, \tilde{R}_m)}{(1+r)^h}, \tag{10}$$

Where $\lambda_h$is the market price of risk (over horizon h).

Plugging the expressions for expected value and covariance from A.2 & A.3 into this valuation equation yields:

$$\begin{aligned} C_0 \quad &= \frac{e^{r\,(h-\tau)}\,C[0,\tau]\ -\ \lambda_h\,e^{\left(\mu_m+\frac{1}{2}\sigma_m^2\right)h+r\,(h-\tau)}(C[\rho\,\sigma_s\,\sigma_m\,\tau,\tau] - C[0,\tau])}{(1+r)^h} \\ &= \frac{C[0,\tau] - \lambda_h\,e^{(\mu_m+\frac{1}{2}\sigma_m^2)\,h}\,(C[\rho\,\sigma_s\,\sigma_m\,\tau,\tau] - C[0,\tau])}{e^{r\,\tau}} \end{aligned} \tag{11}$$

Special Cases

Case 1: If markets are complete (so risk-neutral pricing holds) then $\lambda_h = 0$, $\mu_m + \frac{1}{2}\sigma_m^2 = r$. Thus the valuation formula reduces to

$$C_0 = \frac{C[0,\tau]}{e^{r\,\tau}} \ = \ C_{\mathrm{BS}}(S_0, K, \tau, r, \sigma_s)\,, \tag{12}$$

the standard Black–Scholes model option price.

Case 2: If the planning horizon equals the option maturity (i.e., $h = \tau$), then the valuation equation simplifies (see Equations 20–21 in H&T) to

$$C_0 = \frac{C[0,\tau] - \lambda_\tau\,e^{(\mu_m+\frac{1}{2}\sigma_m^2)\,\tau}\,(C[\rho\,\sigma_s\,\sigma_m\,\tau,\tau] - C[0,\tau])}{e^{r\,\tau}} \tag{13}$$

Here $\lambda_\tau$is defined in terms of parameters μ_m, σ_m, r. (Equation 21 in H&T)

I. Interpretation

- The term $C[0,\tau]$is the Black–Scholes call value under zero correlation adjustment or risk-neutral setting.
- The corrective term involving $\lambda_h e^{(\mu_m+\frac{1}{2}\sigma_m^2)\,h}(C[\rho\,\sigma_s\,\sigma_m\,\tau,\tau] - C[0,\tau])$captures the systematic risk of the call's payoff (via the covariance with the market) and the investor's planning horizon.
- When horizon $h$differs from maturity τ, or when markets are incomplete, this adjustment matters.
- In the limiting case of complete markets and horizon = maturity the adjustment vanishes and one recovers the Black–Scholes price.

J. Option Beta Derivation (Sketch)

Husmann & Todorova (2013) further derive the option's asset pricing beta (β-asset) and the covariance beta (β-cov) and show that under their CAPM-option pricing model, these coincide.

They define the option asset-pricing beta as:

$$\beta_h^a = \frac{E[\,C_h] - e^{r\,h} C_0}{E[\,R_m] - e^{r\,h}} \qquad (14)$$

After detailed derivation they find an expression (their Equation 14) linking the option beta to the underlying asset's beta and the option's sensitivity (via delta) in the limit as $h \to 0$. They show that

$$\lim_{h \to 0} \beta_h^a = \beta_S \cdot \frac{\frac{\partial C}{\partial S}}{\frac{C}{S}}, \qquad (15)$$

Where $\beta_S$is the underlying asset's beta, and $\partial C / \partial S$is the option delta. This replicates the well-known relation in continuous time

Conclusion

In this paper we have revisited the valuation of option contracts through the lens of the CAPM, showing that the familiar risk-neutral, no-arbitrage paradigm can be enriched by equilibrium asset pricing considerations. Our theoretical framework demonstrates that an option's value is not solely determined by underlying volatility and time to maturity, but also by its systematic risk (covariance with the market portfolio) and the investor's planning horizon (which may differ from the contract maturity). In complete markets and when investor horizon equals the option's tenure, the model naturally collapses to the Black–Scholes formula, ensuring internal consistency.

Empirically, the proposed approach suggests that options written on underlyings with higher market betas should exhibit higher required returns and that pricing deviations from standard models (e.g., Black–Scholes) should vary systematically with underlying beta, maturity differences and moneyness. If supported by data, these results imply that practitioners and academics alike should pay attention to the broader asset-pricing context of derivative contracts — not just their volatility and replication cost.

Furthermore, in markets characterized by incomplete trading, heterogeneous investor horizons, and limited liquidity (as is often the case in emerging markets), the CAPM-option framework may offer a valuable complement to standard models. Risk managers should consider option betas and horizon mismatches when hedging and pricing, while researchers may extend the framework to multi-factor settings or non-Gaussian returns.

Nevertheless, limitations must be acknowledged. Estimation of the option's effective beta, measurement of investor horizon, and the assumption of lognormal returns present challenges. The empirical tests may suffer from data limitations, especially in less-liquid markets. Future research could explore empirical implementation across different markets, extend the model to American options or exotic payoffs, and integrate richer return dynamics (jumps, stochastic volatility) within the CAPM option-pricing framework.

In sum, bridging equilibrium asset pricing and derivative valuation enriches both theory and practice. By recognizing that options carry systematic risk and that investor horizon matters, the CAPM-option approach offers a more holistic understanding of option pricing — one that aligns derivatives within the broader architecture of financial markets.

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